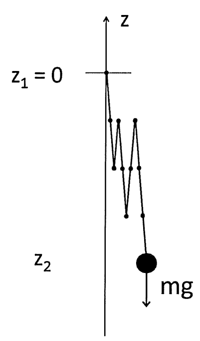
Exam Statistical Physics II

Monday, January 28, 2013

The total number of points is 50. Do not forget to write your name and the student number on the first sheet. Good luck.

1. **Rubber molecule with an attached mass**

   Consider a chain which consists of $N \gg 1$ links each of length a. The links all lie along the vertical z-axis but they may double back on themselves. One end of the chain is fixed at $z_1 = 0$. The second end of the chain is attached to a point-like load of mass m. The weight of the chain is small compared to mg. Describe the system (chain+load) with a microcanonical ensemble and find the dependence of the coordinate z_2 of the second chain end on the temperature T of the chain. [8 points]



   Hint: Denote by $N_\uparrow$ and $N_\downarrow$ the number of links oriented, respectively, up and down. Express the entropy S and the energy E of the system through $N_\uparrow$ and $N_\downarrow$.

2. **Gas of ultrarelativistic particles**

   Consider ideal classical gas of N ultrarelativistic particles with the energy $\varepsilon_{\mathbf{p}} = c|\mathbf{p}|$, where $\mathbf{p}$ is the momentum of the particle and c is the speed of light. The gas occupies volume V and has temperature T.

(a) Show that the single-particle partition function

$$z = \sum_{\mathbf{p}} e^{-\beta \varepsilon_{\mathbf{p}}} = \frac{V}{\pi^2} \left(\frac{k_B T}{\hbar c} \right)^3 .$$

[3 points]

(b) Find the dependence of the free energy F and the entropy S of the gas on temperature, volume and the number of particles. [4 points]

(c) Show that the pressure P and energy E of the gas are related by $PV = \frac{1}{3}E$. [4 points]

(d) Find the ratio of heat capacities at constant pressure and constant volume, $\gamma = \frac{c_P}{c_V}$. [3 points]

Hint:

$$\int_0^\infty dx x^n e^{-x} = n!$$

3. **Specific heat of gases**

The table below shows the constant volume specific heat per molecule, c_v, of three different gases measured at 1 atm and 15 °C.

gas	c_v/k_B
Ar	1.50
NO	2.51
H_2S	3.06

Can you make sense out of these numbers? [8 points]

Hint: What can one conclude about the rotational and vibrational temperatures of these gases?

4. **Expansion of Fermi gas**

The volume V is occupied by N spin-$\frac{1}{2}$ particles of mass m forming ideal Fermi gas at zero temperature. The energy of the particle is $\varepsilon_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m}$.

(a) Show that the energy of the gas is given by

$$E = \frac{3}{5} N \varepsilon_F ,$$

where $\varepsilon_F = \frac{p_F^2}{2m}$ is the Fermi energy and $p_F = \hbar\left(\frac{3\pi^2 N}{V}\right)^{1/3}$ is the Fermi momentum. [3 points]

(b) Show that the pressure of the gas is

$$P = \frac{2}{5}\frac{N}{V}\varepsilon_F.$$

[3 points]

(c) The volume of the container *instantaneously* increases from V to $V' \gg V$. After expansion the gas became *classical.* Find the temperature T' of the gas after the expansion and the ratio of the final and initial pressures, $\frac{P'}{P}$. [6 points]

Hint: If the volume of the container increases very fast, the gas has no time to do work.

5. **Moving to Mars**

As the Earth gradually gets overpopulated, one should start thinking about colonizing other planets of the solar system. Mars, for example, is only ~ 1.5 times farther from the Sun than the Earth. The average temperature of the Earth surface is about 15°C or ~ 288 K. Estimate the average temperature on Mars assuming that both planets are black bodies. [8 points]

Hint: How does the intensity of emission from a black body depend on temperature? Do not worry about the greenhouse effect – Mars atmosphere will also be polluted once people are there.